NOTATION

u, velocity component along x axis; ν, μ, kinematic and dynamic viscosities; ρ, density of liquid; g, projection of the acceleration of free fall onto the x axis; σ, surface tension at the liquid–gas interface; Γ, surface concentration of insoluble surface-active material; h(x), film thickness; $F(\Gamma)$, kinematic function expressing the dependence of the rate of the chemical surface reaction on the concentration of the surface-active material; k, reaction rate constant of dimensions 1/T; C_0, concentration of component of the gas which dissolves in the liquid close to the film surface; D, diffusion coefficient of gas dissolved in the film.

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INFLUENCE OF LONGITUDINAL MIXING ON DISSOLUTION KINETICS OF A POLYDISPERSE SYSTEM OF PARTICLES

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The problem of dissolution of a polydisperse system of particles in a semibounded channel is formulated and investigated with longitudinal mixing in the solid phase taken into account.

To describe processes proceeding in polydisperse systems, methods of the mechanics of heterogeneous media [1, 2] have recently received greater and greater currency. According to [1], utilization of the so-called continuity equations for functions of the particle size distribution density type turns out to be most productive when studying the motion of inclusions interacting with a dispersion medium and hence changing size. Thus on the basis of this approach mathematical models of certain processes of such nature are obtained and investigated in [3, 4].

Let us examine the steady process of dissolution of a polydisperse system of solid inclusions entrained by a fluid flow moving in a semibounded channel. Such a representation can be used if it is assumed that the main particle mass is dissolved without succeeding in reaching the opposite boundary of the channel. Let us take the quasihomogeneity hypothesis [5] that the spacings in which the mixture flow parameters change substantially, are much greater than the particle size and their separation. We will consider the inclusions whose sizes are a continuous random variable, to be sufficiently numerous so that their granulometric composition could be described by a continuous function of the size distribution density type (not normalized to one) that satisfies the continuity equation in the space of their linear dimensions. If the fluctuations of the linear rate of particle dissolution is neglected, then for the developed turbulent flow case, the continuity equation can be written in the form

$$w\frac{\partial f}{\partial l}+\frac{\partial}{\partial r}(fv)-D\frac{\partial^2 f}{\partial l^2}=\delta(l-l_0)\,wf_0(r). \tag{1}$$

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Introduction of a solid phase in the channel is taken into account by the source component in the right side of (1), which indicates that all particles with initial distribution density $f_0(r)$ proceed in the channel section with coordinate $l_0 \neq 0$. The condition of impermeability of the baffle at $l = 0$ for the solid phase and also the condition of no particles at $l \to \infty$ because of their total dissolution up to the time of passing this point and for $r \to \infty$ that results from the properties of the distribution density function are formulated in the form

$$f|_{l=0} - \frac{D}{w}\frac{\partial f}{\partial l}\bigg|_{l=0} = 0; \qquad \lim_{l\to\infty} f(r,\ l) = 0; \qquad \lim_{r\to\infty} f(r,\ l) = 0. \tag{2}$$

We find the linear rate of single particle dissolution from the mass transfer equation on the interface of the solid and liquid phases

$$v(r,\ l) = -\frac{k_w}{3\rho k_v}\beta(r)(c^* - c).$$

According to [6], the form of the functional dependence $\beta(r)$ is due to the magnitude of the relative velocity of the motion of the inclusion and the fluid. We solve the formulated problem for an arbitrary dependence of the mass transfer coefficient on the linear dimension of the particle. In the general case the moving force of the dissolution process $c^* - c$ varies along the channel length while the running concentration c is related to the mass of the substance being dissolved by the conservation equation, which makes the problem (1) and (2) nonlinear and results in mathematical difficulties in its solution. We assume that dissolution occurs in a large volume and the moving force retains a constant value, then the influence of the longitudinal mixing on the granulometric composition of the inclusion can be set up on the basis of an analytic solution of the problem (1) and (2).

Let us give the scale of the particle size r_m and let us select as characteristic length L the extent of the section that a particle will traverse that has a dimension r_m at the beginning of the path as it moves along the channel axis at the mean velocity w until complete dissolution

$$L = \frac{3w\rho k_v}{k_w(c^* - c)}\int_0^{r_m}\frac{dr}{\beta(r)}.$$

Let us go over to dimensionless variables by means of the formulas

$$x = r/r_m,\ y = l/L, \quad y_0 = l_0/L, \quad U(x,\ y) = f(xr_m,\ yL)\,r_m SL. \tag{3}$$

Executing the change of variables

$$z(x) = \int_0^x \frac{d\xi}{\beta_0(\xi)}; \qquad V[z(x),\ y] = \beta_0(x)\,U(x,\ y)\exp\left\{-\frac{\mathrm{Pe}}{2}\left[y + \frac{z(x)}{2}\right]\right\};$$
$$V_0[z(x)] = U_0(x)\,\beta_0(x)\exp\left[-\frac{\mathrm{Pe}}{4}z(x)\right], \tag{4}$$

where

$$\beta_0(x) = \frac{\beta(xr_m)}{r_m}\int_0^{r_m}\frac{dr}{\beta(r)}; \quad \mathrm{Pe} = \frac{wL}{D}; \quad U_0(x) = f_0(xr_m)\,r_m SL,$$

we write the initial equation (1) in the more compact form

$$\frac{\partial V}{\partial z} + \frac{1}{\mathrm{Pe}}\frac{\partial^2 V}{\partial y^2} + \delta(y - y_0)\exp\left(-\frac{\mathrm{Pe}}{2}y\right)V_0(z) = 0, \tag{5}$$

while the boundary conditions (2) take the following form in the new variables (4)

$$\frac{1}{2}V\bigg|_{y=0} - \frac{1}{\mathrm{Pe}}\frac{\partial V}{\partial y}\bigg|_{y=0} = 0; \ \lim_{y\to\infty} V(z,\ y) = 0; \ \lim_{z\to\infty} V(z,\ y) = 0. \tag{6}$$

It is possible to go over to the true function of the distribution density type $f(r,\ l)$ by means of the function

$$f(r,\ l) = \frac{V[z(r/r_m),\ l/L]}{r_m SL\beta_0(r/r_m)}\exp\left[\frac{\mathrm{Pe}}{4}(2l/L + r/r_m)\right].$$

Solving the problem (5) and (6) by using the Fourier transform [7] and returning to the dimensionless variables (3) by using the relationships (4), it can be shown that the solution of the initial problem (1) and (2) is determined by the formula

$$U(x,\ y)=\frac{1}{2\beta_0(x)}\sqrt{\frac{\mathrm{Pe}}{\pi}}\int_x^\infty\frac{U_0(\xi)}{t}\left\{\exp\left[-\mathrm{Pe}\,y_0-\frac{\mathrm{Pe}}{4}\left(t-\frac{y+y_0}{t}\right)^2\right]+\exp\left[-\frac{\mathrm{Pe}}{4}\left(t-\frac{y-y_0}{t}\right)^2\right]\right\}d\xi- \tag{7}$$

$$-\frac{\mathrm{Pe}}{2\beta_0(x)}\exp(\mathrm{Pe}\,y)\int_x^\infty U_0(\xi)\,\mathrm{erfc}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(t+\frac{y+y_0}{t}\right)\right]d\xi,$$

where

$$t=\sqrt{z(\xi)-z(x)}. \tag{8}$$

To simplify the further interpretation of the solution obtained, it is convenient to execute a change of variable of integration in (7). Using (8), we obtain

$$U(x,\ y)=\frac{1}{\beta_0(x)}\sqrt{\frac{\mathrm{Pe}}{\pi}}\int_0^\infty W(t,\ x)\left\{\exp\left[-\mathrm{Pe}\,y_0-\frac{\mathrm{Pe}}{4}\left(t-\frac{y+y_0}{t}\right)^2\right]+\right. \tag{9}$$

$$\left.+\exp\left[-\frac{\mathrm{Pe}}{4}\left(t-\frac{y-y_0}{t}\right)^2\right]\right\}dt-\frac{\mathrm{Pe}\exp(\mathrm{Pe}\,y)}{2\beta_0(x)}\int_0^\infty W(t,\ x)\,t\,\mathrm{erfc}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(t+\frac{y+y_0}{t}\right)\right]dt.$$

Here

$$W(t,\ x)=\beta_0\{z^{-1}[t^2+z(x)]\}\,U_0\{z^{-1}[t^2+z(x)]\},$$

where z^{-1} is the inverse function to z(x). By using (9) we compute the specific solid phase particle flux through an arbitrary channel section

$$U(x,\ y)-\frac{1}{\mathrm{Pe}}\frac{\partial U}{\partial y}=\frac{1}{2\beta_0(x)}\sqrt{\frac{\mathrm{Pe}}{\pi}}\int_0^\infty W(t,\ x)\left\{\exp\left[-\frac{\mathrm{Pe}}{4}\left(t-\frac{y-y_0}{t}\right)^2\right]\left(1+\frac{y-y_0}{t^2}\right)-\right. \tag{10}$$

$$\left.-\exp\left[\mathrm{Pe}\,y-\frac{\mathrm{Pe}}{4}\left(t+\frac{y+y_0}{t}\right)^2\right]\left(1-\frac{y+y_0}{t^2}\right)\right\}dt.$$

The right side of (10) has discontinuity at the point of solid phase insertion with coordinate y_0. If the function W(t, x) is bounded in t in the interval (0, ∞) and is continuous at the point t = 0, then it can be shown that the magnitude of the jump at the point of discontinuity is $U_0(x)$.

Graphs of the function U(x, y) constructed by means of (7) for $f_0(r) = N_0\delta(r - r_m)$ or in dimensionless form $U_0(x) = N_0SL\delta(x - 1)$ for the case $\beta_0(x)$ = const = 1 are represented in Fig. 1, from which it follows that an increase in the dispersion of the inclusions occurs under the effect of longitudinal mixing with distance from the point of solid phase insertion. A graph of the function $f_0(r)$ corresponding to monodisperse composition of the particles delivered to the channel entrance is marked by the dashed line.

The solution (7) or (9) obtained is suitable for the whole range of Pe numbers, however, calculations using these formulas for large values of Pe can yield significant errors because of the nonuniform convergence of the integral in the parameter Pe, in which connection we present an asymptotic expansion of (9) obtained for large Pe numbers by the Laplace method [8] in the form

$$U(x,\ y)=\frac{2}{\beta_0(x)}\sum_{n=0}^{\infty}\frac{(2n)!}{\mathrm{Pe}^n\,n!}\left\{\exp(-\mathrm{Pe}\,y_0)\left(A_{2n}-\frac{\mathrm{Pe}}{2}E_{2n}\right)+H(y_0-y)\exp[-\mathrm{Pe}(y_0-y)]B_{2n}+H(y-y_0)C_{2n}\right\}, \tag{11}$$

where A_i, B_i, C_i, E_i are coefficients of the Maclauren series expansion in powers of τ for the functions

$$\frac{1}{2}\left(1+\frac{\tau}{2\sqrt{\frac{\tau^2}{4}+y_0+y}}\right)W\left[\left(\frac{\tau}{2}+\sqrt{\frac{\tau^2}{4}+y_0+y}\right),\ x\right];$$

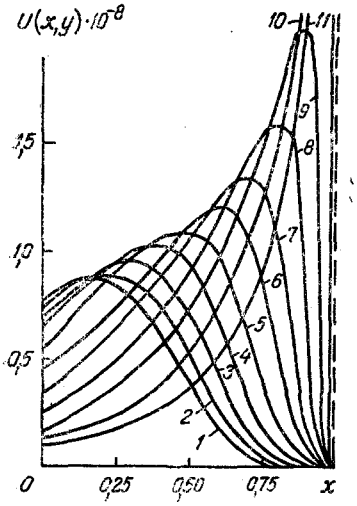

Fig. 1. Dimensionless function of the particle size distribution density type at the point y = 1 for Pe = 10 and $N_0 = 0.9 \cdot 10^8$. Corresponding to the curves 1-11 are the following values of the dimensionless coordinate of the point of solid phase insertion y_0 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1.

$$\frac{1}{2}\left(1+\frac{\tau}{2\sqrt{\frac{\tau^2}{4}+y_0-y}}\right)W\left[\left(\frac{\tau}{2}+\sqrt{\frac{\tau^2}{4}+y_0-y}\right),\ x\right];$$

$$\frac{1}{2}\left(1+\frac{\tau}{2\sqrt{\frac{\tau^2}{4}+y-y_0}}\right)W\left[\left(\frac{\tau}{2}+\sqrt{\frac{\tau^2}{4}+y-y_0}\right),\ x\right];$$

$$\left[1-\frac{y+y_0}{\frac{\tau^2}{4}+y+y_0+\frac{\tau}{2}\sqrt{\frac{\tau^2}{4}+y+y_0}}\right]\int_0^{\frac{\tau}{2}+\sqrt{\frac{\tau^2}{4}+y_0+y}} W(\xi,\ x)\,d\xi.$$

As Pe → ∞, by limiting ourselves to the first terms in the expansion (11) we obtain

$$U(x,\ y)=\frac{1}{\beta_0(x)}H(y-y_0)\,W\left(\sqrt{y-y_0},\ x\right). \tag{12}$$

The formulas (7) and (9) for functions of the particle size distribution density type permit computation of their concentration profile along the channel length, which is determined by the initial zeroth order moment for the function f(r, l), for which the expression in the variables (3) has the form

$$\mu_0(y)=\int_0^\infty U(x,\ y)\,dx=\frac{1}{2}\int_0^\infty U_0(\xi)\left\{\left[1+\operatorname{erf}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(q-\frac{y-y_0}{q}\right)\right]-\right.\right.$$

$$\left.-\exp\left[\mathrm{Pe}\,(y-y_0)\right]\operatorname{erfc}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(q+\frac{y-y_0}{q}\right)\right]\right]H(y-y_0)+$$

$$+\left[\exp\left[-\mathrm{Pe}\,(y_0-y)\right]+\exp\left[-\mathrm{Pe}\,(y_0-y)\right]\operatorname{erf}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(q-\frac{y_0-y}{q}\right)\right]-\right. \tag{13}$$

$$\left.-\operatorname{erfc}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(q+\frac{y_0-y}{q}\right)\right]\right]H(y_0-y)-\mathrm{Pe}\,(y+y_0+q^2)\exp(\mathrm{Pe}\,y)\times$$

$$\times\operatorname{erfc}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(q+\frac{y+y_0}{q}\right)\right]+2q\sqrt{\frac{\mathrm{Pe}}{\pi}}\exp\left[\mathrm{Pe}\,y-\frac{\mathrm{Pe}}{4}\left(q+\frac{y+y_0}{q}\right)^2\right]\Bigg\}\,d\xi,$$

where $q=\sqrt{z(\xi)}$.

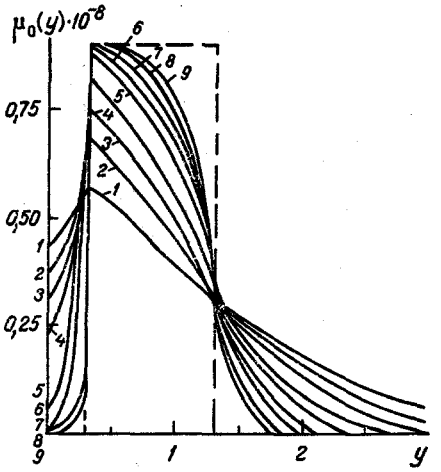

Fig. 2. Dependences of the dimensionless zeroth moment of the function on the coordinate y for $y_0 = 0.3$ and $N_0 = 0.9 \cdot 10^8$ and different values of Pe: 1) Pe = 1, 2) 2, 3) 3, 4) 5, 5) 10, 6) 14, 7) 28, 8) 37, 9) 41.

In the ideal displacement regime (12) the expression (13) becomes

$$\mu_0(y) = H(y - y_0) \int_{z^{-1}(y-y_0)}^{\infty} U_0(\xi)\, d\xi. \tag{14}$$

Presented in Fig. 2 are graphs of the function (13) for the case considered above. The dashed curve corresponds to the limit expression (14). Instantaneous dissolution of all particles of a narrow fraction delivered to the channel entrance and achieving the zeroth dimension at this point occurs at the point of intersection of all the graphs with abscissa $y = y_0 + 1 = 1.3$ in the ideal displacement regime. As is seen from the figure, a diminution in the number Pe results in an increase in the length of the effective section of the channel needed to dissolve the main part of the inclusions.

The relationship (13) should satisfy the conservation equation for the integral balance of the number of particles for an arbitrary channel section, that can be obtained from (1) by integrating it with respect to r between the limits (0, ∞). In the dimensionless variables it takes the form

$$\frac{d\mu_0}{dy} - \frac{1}{\mathrm{Pe}} \frac{d^2\mu_0}{dy^2} + \lim_{x \to 0} (U\beta_0) - \delta(y - y_0)\, \mu_0^* = 0 \tag{15}$$

with boundary conditions that result from the conditions (2) in the form

$$\mu_0|_{y=0} - \frac{1}{\mathrm{Pe}} \frac{d\mu_0}{dy}\bigg|_{y=0} = 0; \quad \lim_{y \to \infty} \mu_0(y) = 0,$$

where

$$\mu_0^* = \int_0^{\infty} U_0(\xi)\, d\xi.$$

The first source component in (15), which appears in a natural manner upon integration of the initial problem is the number of particles being dissolved completely per unit volume of suspension per unit time in a given channel section. This component is a result of the polydispersity of the particle composition, that results in the fact that in each channel section inclusions of all the dimensional groups, including even particles of vanishingly small size which are continuously driven out of the integral balance of particles during dissolution, are present with a certain probability. By calculating the limits we obtain the following dimensionless expression for the intensity of inclusion "disappearance":

$$I(y) = \lim_{x \to 0} (U\beta_0) = \frac{1}{2} \sqrt{\frac{\mathrm{Pe}}{\pi}} \int_0^{\infty} \frac{U_0(\xi)}{q} \left\{ \exp\left[-\mathrm{Pe}\, y_0 - \right. \right.$$

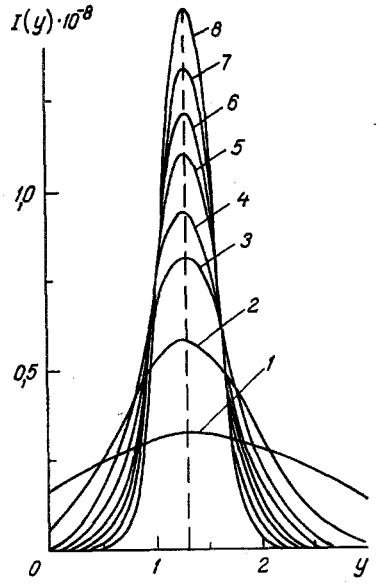

Fig. 3. Dependence of the intensity of particle "disappearance" on the coordinate y for the initial distribution $U_0(x) = N_0 SL\delta(x-1)$, $y_0 = 0.3$, $N_0 = 0.9 \cdot 10^8$ and different values of Pe: 1) Pe = 1, 2) 5, 3) 10, 4) 14, 5) 19, 6) 23, 7) 28, 8) 37.

$$-\frac{\mathrm{Pe}}{4}\left(q-\frac{y+y_0}{q}\right)^2\Big]+\exp\left[-\frac{\mathrm{Pe}}{4}\left(q-\frac{y-y_0}{q}\right)^2\right]\Big\}d\xi- \\ -\frac{\mathrm{Pe}}{2}\exp(\mathrm{Pe}\,y)\int_0^\infty U_0(\xi)\,\mathrm{erfc}\left[\frac{\sqrt{\mathrm{Pe}}}{2}\left(q+\frac{y+y_0}{q}\right)\right]d\xi, \tag{16}$$

or for case (12)

$$I(y) = H(y-y_0)\,\beta_0\,|z^{-1}(y-y_0)]\,U_0\,|z^{-1}(y-y_0)|. \tag{17}$$

Graphs of the function (16) are displayed in Fig. 3 for the example under consideration, from which it follows that as the number Pe grows the domain in which disappearance of the main mass of particles occurs contracts and in the limit case of ideal displacement (17) the whole domain is concentrated into one point $y = y_0 + 1 = 1.3$ and the function I(y) degenerates into a δ-function denoted by the dashed curve.

In the steady-state regime the number of particles delivered to the entrance per unit time should equal the number of particles being dissolved completely in the channel in that same time, or in dimensionless form

$$\int_0^\infty I(y)\,dy = \int_0^\infty U_0(x)\,dx. \tag{18}$$

It can be seen by direct calculations that in both the main (16) and the limit (17) cases this equality is conserved.

The following expression is valid for the total number of particles found in the channel

$$G = \int_0^\infty \mu_0(y)\,dy = \int_0^\infty U_0(x)\,z(x)\,dx, \tag{19}$$

which shows that delay G is determined completely by the rate of access and the rate of dissolution of the inclusions and is independent of the mixing regime in the channel and the coordinate of the solid phase insertion.

Retention of the conservation conditions (18) and (19) indicates admissibility of the assumptions made in formulation of the problem, and correctness of the solutions obtained.

Utilization of the distribution density type function f(r, l) and the continuity equation for it permits estimation of the physical meaning and the structure of the source term in (13)

for the particle concentration, whose appearance is due to polydispersity of the inclusions. Analysis of (7), (11), and (12) as well as Fig. 2 indicate that as the number Pe increases the particle size distribution density type function is narrowed and the length of the effective channel section is diminished.

Taking account of the influence of longitudinal mixing on the granulometric composition of particles being dissolved can contribute to refinement of the mathematical models of a number of processes with the participation of polydisperse systems.

Remark. The function $f(r, l)$ is defined as follows: $f(r, l) = d/dr\ N(r, l)$, where $dN(r, l)$ is the number of particles per unit volume of suspension, per size interval $(r, r + dr)$. where dr is generally understood not mathematically but physically as an infinitesimal size range, i.e., a sufficiently small range as compared with the particle sizes but still containing a sufficiently large quantity of them at the same time so that an insignificant change of the number of particles in it does not, in practice, change the function $f(r, l)$ as compared with its absolute value. This latter circumstance permits extension of the domain of values of this function to the whole real half-line and to apply the operations of differentiation and integration to it in the ordinary sense.

NOTATION

$f(r, l)$, $f_0(r)$, particle distribution density type functions in the dimension r in a section with coordinate l and at the channel entrance; $v(r, l)$, linear rate of dissolution of a single particle; w, mean velocity of the solid phase flux; D, flux longitudinal mixing coefficient; S, channel section area; δ, Dirac function; c and c*, concentrations of the dissolved substance in the flux core and on the solid particle surface; $\beta(r)$, mass transfer coefficient; k_w and k_v, surface and bulk coefficients of particle shape; L, channel length scale; x, y, $U(x, y)$, $U_0(x)$, dimensionless variables; ξ, interior variable of integration; Pe, Peclet number; H, Heaviside function; $\mu_0(y)$, zeroth-order initial moment of the dimensionless function of the particle size distribution density type; N_0, number of particles per unit volume of suspension at the channel entrance; I(y), intensity of particle "disappearance"; ρ, solid phase density; r_m, scale of particle size.

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